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Polynomial Liénard systems with a nilpotent global center

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Abstract
A center for a differential system $\dot{\mathbf{x}} = f(\mathbf{x})$ in $\mathbb{R}^2$ is a singular point p having a neighborhood U such that $U \setminus \{p\}$ is filled with periodic orbits. A global center is a center p such that $\mathbb{R}^2 \setminus \{p\}$ is filled with periodic orbits. There are three kinds of centers, the centers p such that the Jacobian matrix $Df(p)$ has purely imaginary eigenvalues, the nilpotent centers p such that $Df(p)$ is a nilpotent matrix, and the degenerate centers p such that the matrix $Df(p)$ is the zero matrix. For the first class of centers there are several works studying when such centers are global. As far as we know there are no works for studying the nilpotent global centers. One of the most studied classes of differential systems in $\mathbb{R}^2$ are the polynomial Liénard differential systems. The objective of this paper is to study the nilpotent global centers of the polynomial Liénard differential systems.

Keywords Center · Global center · Periodic orbits · Nilpotent singularity

Mathematics Subject Classification 34C05

1 Introduction and statement of the main results

The study of the global centers of the polynomial differential systems was initiated by Conti and their collaborators, see [4, 5, 7]. There were some previous articles on the global centers before the work of Conti but without this name and for differential equations which were not polynomial. It is known that the polynomial differential systems with a global center have odd degree, see [7, 11]. Also it is known that the unique polynomial differential systems which have a rigid global center are the linear differential centers, see [4]. A center p is *rigid*

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